Maths skills

1 Measurements

1.1 Base and derived SI units

Units are defined so that, for example, every scientist who measures a mass in kilograms uses the same size for the kilogram and gets the same value for the mass. Scientific measurement depends on standard units – most are *Système International* (SI) units. Every measurement must give the unit to have any meaning. You should know the correct unit for physical quantities.

Base units

Physical quantity	Unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s

Physical quantity	Unit	Symbol
electric current	ampere	A
temperature difference	Kelvin	K
amount of substance	mole	mol

Derived units

Example:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

If a car travels 2 metres in 2 seconds:

$$\text{speed} = \frac{2\,\text{metres}}{2\,\text{seconds}} = 1\frac{\text{m}}{\text{s}} = 1\,\text{m/s}$$

This defines the SI unit of speed to be 1 metre per second (m/s), or 1 m s^{-1} ($s^{-1} = \frac{1}{s}$).

Practice questions

1 Complete this table by filling in the missing units and symbols.

Physical quantity	Equation used to derive unit	Unit	Symbol and name (if there is one)
frequency	period^{-1}	s^{-1}	Hz, hertz
volume	length^3		–
density	mass ÷ volume		–
acceleration	velocity ÷ time		–
force	mass × acceleration		
work and energy	force × distance		

1.2 Significant figures

When you use a calculator to work out a numerical answer, you know that this often results in a large number of decimal places and, in most cases, the final few digits are 'not significant'. It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have, too few and you are not showing the precision and care required in scientific analysis.

Numbers to 3 significant figures (3 s.f.):

3.62   25.4   271   0.0147   0.245   39 400

(notice that the zeros before the figures and after the figures are *not* significant – they just show you how large the number is by the position of the decimal point).

Numbers to 3 significant figures where the zeros *are* significant:

207   4050   1.01 (any zeros between the other significant figures *are* significant).

Standard form numbers with 3 significant figures:

9.42×10^{-5} 1.56×10^{8}

If the value you wanted to write to 3.s.f. was 590, then to show the zero was significant you would have to write:

590 (to 3.s.f.) or 5.90×10^{2}

Practice questions

2 Give these measurements to 2 significant figures:

a 19.47 m   **b** 21.0 s   **c** 1.673×10^{-27} kg   **d** 5 s

3 Use the equation:

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

to calculate the resistance of a circuit when the potential difference is 12 V and the current is 1.8 mA. Write your answer in kΩ to 3 s.f.

1.3 Uncertainties

When a physical quantity is measured there will always be a small difference between the measured value and the true value. How important the difference is depends on the size of the measurement and the size of the uncertainty, so it is important to know this information when using data.

There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement and the resolution of the measuring instrument (i.e. the size of the scale divisions).

For example, a length of 6.5 m measured with great care using a 10 m tape measure marked in mm would have an uncertainty of 2 mm and would be recorded as 6.500 ± 0.002 m.

It is useful to quote these uncertainties as percentages.

For the above length, for example,

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measurement}} \times 100$$

$\text{percentage uncertainty} = \dfrac{0.002}{6.500} \times 100\% = 0.03\%$. The measurement is 6.500 m ± 0.03%.

Values may also be quoted with absolute error rather than percentage uncertainty, for example, if the 6.5 m length is measured with a 5% error,

the absolute error = 5/100 × 6.5 m = ±0.325 m.

Practice questions

4 Give these measurements with the uncertainty shown as a percentage (to 1 significant figure):

a 5.7 ± 0.1 cm **b** 450 ± 2 kg **c** 10.60 ± 0.05 s **d** 366 000 ± 1000 J

5 Give these measurements with the error shown as an absolute value:

a 1200 W ± 10% **b** 330 000 Ω ± 0.5%

6 Identify the measurement with the smallest percentage error. Show your working.

A 9 ± 5 mm **B** 26 ± 5 mm **C** 516 ± 5 mm **D** 1400 ± 5 mm