

3 Resolving vectors

3.1 Vectors and scalars

Vectors have a magnitude (size) and a direction. Directions can be given as points of the compass, angles or words such as forwards, left or right. For example, 30 mph east and 50 km/h north-west are velocities.

Scalars have a magnitude, but no direction. For example, 10 m/s is a speed.

Practice questions

- State whether each of these terms is a vector quantity or a scalar quantity: density, temperature, electrical resistance, energy, field strength, force, friction, frequency, mass, momentum, power, voltage, volume, weight, work done.
- For the following data, state whether each is a vector or a scalar: 3 ms^{-1} , $+20 \text{ ms}^{-1}$, 100 m NE, 50 km, -5 cm , 10 km S 30° W, $3 \times 10^8 \text{ ms}^{-1}$ upwards, 273°C , 50 kg, 3 A.

3.2 Drawing vectors

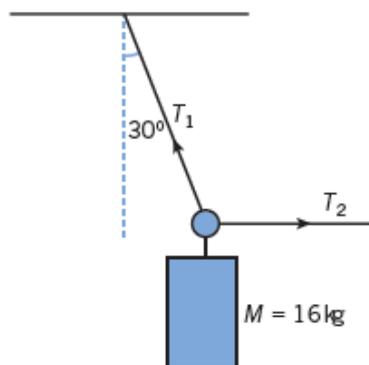
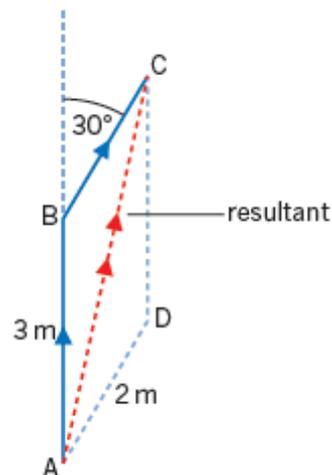
Vectors are shown on drawings by a straight arrow. The arrow starts from the point where the vector is acting and shows its direction. The length of the vector represents the magnitude.

When you add vectors, for example two velocities or three forces, you must take the direction into account.

The combined effect of the vectors is called the resultant.

This diagram shows that walking 3 m from A to B and then turning through 30° and walking 2 m to C has the same effect as walking directly from A to C. AC is the resultant vector, denoted by the double arrowhead.

A careful drawing of a scale diagram allows us to measure these. Notice that if the vectors are combined by drawing them in the opposite order, AD and DC, these are the other two sides of the parallelogram and give the same resultant.



Practice question

- Two tractors are pulling a log across a field. Tractor 1 is pulling north with force 1 = 5 kN and tractor 2 is pulling east with force 2 = 12 kN. By scale drawing, determine the resultant force.

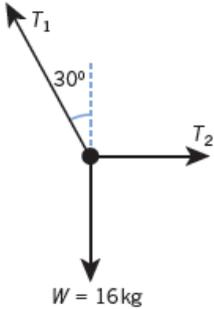
3.3 Free body force diagrams

To combine forces, you can draw a similar diagram to the one above, where the lengths of the sides represent the magnitude of the force (e.g., 30 N and 20 N). The third side of the triangle shows us the magnitude and direction of the resultant force.

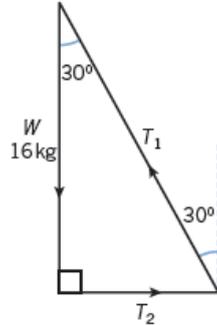
When solving problems, start by drawing a free body force diagram. The object is a small dot and the forces are shown as arrows that start on the dot and are drawn in the direction of the force. They don't have to be to scale, but it helps if the larger forces are shown to be larger. Look at this example.

A 16 kg mass is suspended from a hook in the ceiling and pulled to one side with a rope, as shown on the right. Sketch a free body force diagram for the mass and draw a triangle of forces.

Free body force diagram



Triangle of forces



Notice that each force starts from where the previous one ended and they join up to form a triangle with no resultant because the mass is in equilibrium (balanced).

Practice questions

- 4 Sketch a free body force diagram for the lamp (**Figure 1**, below) and draw a triangle of forces.
- 5 There are three forces on the jib of a tower crane (**Figure 2**, below). The tension in the cable T , the weight W , and a third force P acting at X . The crane is in equilibrium. Sketch the triangle of forces.

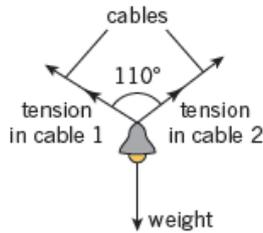


Figure 1

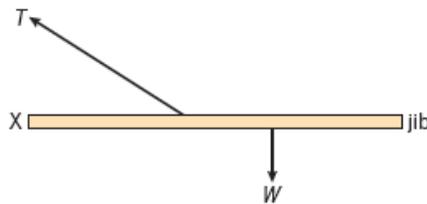


Figure 2

3.4 Calculating resultants

When two forces are acting at right angles, the resultant can be calculated using Pythagoras's theorem and the trig functions: sine, cosine, and tangent.

For a right-angled triangle as shown:

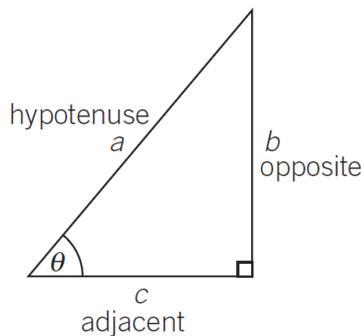
$$h^2 = o^2 + a^2$$

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

(soh-cah-toa).



Practice questions

- 6 **Figure 3** shows three forces in equilibrium.
Draw a triangle of forces to find T and α .
- 7 Find the resultant force for the following pairs of forces at right angles to each other:
a 3.0 N and 4.0 N b 5.0 N and 12.0 N

Figure 3