Bridging GCSE to A-level Maths

Task 1
Recap and review of important GCSE topics

1. Complete Section A (20 questions) over this two week period prior to week 37.
2. If there any topics you need to revise then see the following page for a set of video links.
3. Message Mr Loney on Teams if you need any help during this period, along with a photo of the question/working out.
4. When you are finished and fully happy, please send a photo(s) to Mr Loney on Teams (this should be prior to the start of week 37).

Video links for GCSE revision

1. Completing the Square - https://youtu.be/yO5u7QiqxF4
2. Solving Quadratic Inequalities - https://youtu.be/tTyOlCwgBLY
3. Solving Simultaneous Equations - https://youtu.be/FcYeDDlgNMs
4. Quadratic Simultaneous Equations - https://youtu.be/KSS0nLu5PnE
5. Factorising Quadratics - https://youtu.be/yBI-TZ1myf8
6. Surds - https://youtu.be/zS0B2pfEKUs
7. Surds (Rationalising the Denominator) - https://youtu.be/-No8eFx16T4
8. Negative and Fractional Indices - https://youtu.be/UCcW2ImvTVM
9. Equation of a line - https://youtu.be/-bKW-bgBJRk
10. Distance between two points - https://youtu.be/bs6ODTGktDg
11. Cosine Rule - https://youtu.be/ud8fUf7Wlzc

Task 2
Research into an A-level topic

Algebraic long division (or polynomial long division) is an important and useful topic we study early on in the A-level course. It is a reliable method by which we can factorise expressions that are cubic, quartic, and beyond.

On the next few slides, you will find:

a set of video resources, written examples, and some lovely questions for you to get some practice.

You should aim to produce good quality written notes, containing several examples and clear annotations of what is happening.

Task 2
Research into an A-level topic

Videos

https://youtu.be/-MhwAY83y7g – Good to start with.

https://www.youtube.com/watch?v=A_S1YcVsO80 – Very clear explanations (with a **really good** example around 8:32).

https://www.youtube.com/watch?v=smsKMWf8ZCs – Very good to start with.

https://www.youtube.com/watch?v=RPXMBIFG_W4 – One basic and **one more advanced example** that you won't have seen above.

https://www.youtube.com/watch?v=4u8_AMacu-Y – A slightly different look to the method, only watch this one if the others have been confusing.

Written Example

https://revisionmaths.com/advanced-level-maths-revision/pure-maths/algebra/algebraic-long-division

Task 2
Research into an A-level topic

Questions

Q1. Divide $x^3 + 2x^2 - 17x + 6$ by $(x - 3)$.

Q2. Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

b $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

c $x^3 - x^2 + x + 14$ by $(x + 2)$

d $x^3 + x^2 - 7x - 15$ by $(x - 3)$

Q3. Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

b $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

c $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$

d $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

All answers/solutions will be at the very end.

Task 3
Research into an A-level topic

The Factor Theorem

But what if we don't know what we need to divide by? It's easy to work out $91 \div 7$, but what if we didn't know if 7 was a factor of 91?

This is where the Factor Theorem comes in, to help us identify a factor of a polynomial (polynomials are expressions that only have whole number powers – like quadratics and cubics).

On the next few slides, you will again find a list of video resources and some questions to practice. You will then be able to decide if $x + 3$ is a factor of $4x^3 + 3x^2 - 10x + 7$ without needing to carry out the long division (hint – it isn't).

Task 3
Research into an A-level topic

Videos

https://www.youtube.com/watch?v=b88DwALjFdw – Good to start with.

https://www.youtube.com/watch?v=lyMwX8_QZIc - Good to start with.

https://www.youtube.com/watch?v=GitkhIGwN-k – Once you have the idea of the factor theorem, this is great for seeing *why* it is useful (from 4:10)

https://www.youtube.com/watch?v=RtWNnK-m6pE – Well worth watching for some harder ones.

https://www.youtube.com/watch?v=BNTAAZasEAg – CorbettMaths is great

Again, the goal is to create a comprehensive set of notes consisting of examples with clear annotations about what is happening, and anything else that is important that you have heard from the videos.

Task 3
Research into an A-level topic

Questions

Q1. Use the factor theorem to show that:

a $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

b $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

c $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$.

Q2. Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely.

Q3. Show that $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$ and hence factorise the expression completely.

Q4. Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a.

Q5. Given that $(x - 1)$ and $(x + 1)$ are factors of $px^3 + qx^2 - 3x - 7$, find the values of p and q.

If you get a pair of simultaneous equations, you're on the right track.

Algebraic Long Division

Q1.

$$
\begin{array}{r}
x^2 + 5x - 2 \\
x - 3 \overline{)\, x^3 + 2x^2 - 17x + 6} \\
\underline{x^3 - 3x^2} \\
5x^2 - 17x \\
\underline{5x^2 - 15x} \\
-2x + 6 \\
\underline{-2x + 6} \\
0
\end{array}
$$

So $\dfrac{x^3 + 2x^2 - 17x + 6}{x - 3} = x^2 + 5x - 2$

Q2.

a $(x + 1)(x^2 + 5x + 3)$ **b** $(x + 4)(x^2 + 6x + 1)$

c $(x + 2)(x^2 - 3x + 7)$ **d** $(x - 3)(x^2 + 4x + 5)$

Q3.

a $(x + 4)(6x^2 + 3x + 2)$ **b** $(x + 2)(4x^2 + x - 5)$

c $(x + 3)(2x^2 - 2x - 3)$ **d** $(x - 6)(2x^2 - 3x - 4)$

The Factor Theorem

Q1. **a** $f(1) = 0$ **b** $f(-3) = 0$ **c** $f(4) = 0$

Q2. $(x - 1)(x + 3)(x + 4)$

Q3. $(x + 1)(x + 7)(x - 5)$

Q4. $f(x) = 5x^3 - 9x^2 + 2x + a$

$f(1) = 0$

$5(1)^3 - 9(1)^2 + 2(1) + a = 0$

$5 - 9 + 2 + a = 0$

$a = 2$

Q5. $f(x) = px^3 + qx^2 - 3x - 7$

$f(1) = 0$

$p(1)^3 + q(1)^2 - 3(1) - 7 = 0$

$p + q - 3 - 7 = 0$

$p + q = 10$ **(1)**

$f(-1) = 0$

$p(-1)^3 + q(-1)^2 - 3(-1) - 7 = 0$

$-p + q + 3 - 7 = 0$

$-p + q = 4$ **(2)**

(1) + (2):

$2q = 14$

$q = 7$

Substituting in **(1)**:

$p + 7 = 10$

$p = 3$

So $p = 3$, $q = 7$

What do I do now?

When you are finished with your notes and have completed the questions from ‘algebraic long division’ and ‘the factor theorem’:

1. Send a copy of your notes and marked questions to Mr Loney on Teams.
2. Check and correct any of the questions from the GCSE recap and review that you got wrong, if you haven’t already.
3. Complete the assignment on DrFrostMaths that has been set for you – covering algebraic long division and the factor theorem.
4. Additionally, there is a bonus task on the following slides that you could choose to do – Further Maths students should research this too.

Bonus Topic – Exponentials and Logarithms

Dealing with a question like $x^3 = 64$ is rather easy, we can just cube root and get an answer of $x = 4$. The problem comes when we have a problem like $2^x = 64$, how are we meant to find the value of x?

We could use trial and error to find $x = 6$, but you will see new notation and methods which will give us a *reliable method of solving* exponential equations over the next few slides.

Bonus Topic – Exponentials and Logarithms

Videos

https://youtu.be/N6D6y5U0Hv8 - Introduction to logarithms.

https://www.youtube.com/watch?v=F492MeO74fE – Introduction to logarithms.

https://www.youtube.com/watch?v=fRjUvoRsig0 – A more in depth look at what logarithms are and how to use them.

https://youtu.be/Dwxx7V13GfI - Rules of logarithms (make sure you watch this after you become confident with how logarithms work)

https://www.youtube.com/watch?v=rQOn7PKCmmg - Rules of logarithms (again, watch this after you are more confident in how logarithms work).

Bonus Topic – Exponentials and Logarithms

Questions

https://www.mathsgenie.co.uk/resources/as-pure-exponentials-and-logs.pdf
Please ignore Q6, Q9, Q10, Q11, Q12, Q13 (unless you fancy a further challenge with **e)**

Q1. Rewrite using a logarithm.

a $4^4 = 256$ **b** $3^{-2} = \frac{1}{9}$ **c** $10^6 = 1\,000\,000$

d $11^1 = 11$ **e** $(0.2)^3 = 0.008$

Q2. Without using a calculator, find the value of

a $\log_2 8$ **b** $\log_5 25$ **c** $\log_{10} 10\,000\,000$ **d** $\log_{12} 12$

Q3. Write as a single logarithm, then simplify your answer.

a $\log_2 40 - \log_2 5$ **b** $\log_6 4 + \log_6 9$ **c** $2\log_{12} 3 + 4\log_{12} 2$

d $\log_8 25 + \log_8 10 - 3\log_8 5$ **e** $2\log_{10} 2 - (\log_{10} 5 + \log_{10} 8)$

Bonus Topic – Exponentials and Logarithms

Solutions

https://www.mathsgenie.co.uk/resources/as-pure-exponentials-and-logsans.pdf

Q1. **a** $\log_4 256 = 4$ **b** $\log_3 \frac{1}{9} = -2$
c $\log_{10} 1\,000\,000 = 6$ **d** $\log_{11} 11 = 1$
e $\log_{0.2} 0.008 = 3$

Q2. **a** 3 **b** 2 **c** 7 **d** 1

Q3. **a** $\log_2 8 = 3$ **b** $\log_6 36 = 2$ **c** $\log_{12} 144 = 2$
d $\log_8 2 = \frac{1}{3}$ **e** $\log_{10} 10 = 1$